**Series Notes**

Let *f* be a function with derivatives of all orders and with values that are given approximately by the fourth-degree Taylor polynomial

.

a. Find the approximate value of *f*(2.6). What assumption must you make about 2.6 for this approximation to be valid?

b. Use the pattern for equating derivatives to find .

c. Find a fourth degree Taylor Polynomial for , and use it to find an approximation for *g*(1) assuming the series converges if *x* = 1.

d. Use the second derivative test to show that function *g* in part c has a local maximum at *x* = 0.

e. Find a fifth degree Taylor polynomial for

**Solution:**

a. = 5.886272

You must assume that the series converges if *x* = 2.6.

b.

c.

d. By equating derivatives, is a local maximum because the tangent is horizontal and the graph is concave down at *x* = 0.

e.

**Exercises**

I. Derive a power series for the given function. Write enough terms of the series to show the pattern. Also write the general term.

1. 2.

3. 4.

5. 6.

7. 8.

9. 10.

II. Write a Taylor polynomial of the given degree for the function described.

11. Fourth degree Taylor Polynomial expanded about *x* = 2, if and .

12. Fifth degree Taylor Polynomial expanded about *x* = -1, if and

III. Solve the following…

13. Let *f* be a function with derivatives of all orders and with values that are given approximately by the fourth degree polynomial .

a. Find the approximate value of *f*(.4). What assumption must you make about .4 for this approximation to be valid?

b. Use the pattern for equating derivatives to find .

c. Find a sixth degree Taylor Polynomial for , and use it to find an approximation for *g*(1) assuming the series converges if *x* = 1.

d. Use the second derivative test to determine whether g(0) in part c is a local maximum or local minimum.

e. Find a seventh degree Taylor polynomial for

14. Let *f* be a function with derivatives of all orders and with values that are given approximately by the fourth degree polynomial .

a. Find the approximate value of *f*(1). What assumption must you make about .4 for this approximation to be valid?

b. Use the pattern for equating derivatives to find .

c. Find a sixth degree Taylor Polynomial for , and use it to find an approximation for *g*(1) assuming the series converges if *x* = 1.

d. Use the second derivative test to determine whether g(0) in part c is a local maximum or local minimum.

e. Find a fifth degree Taylor polynomial for

For problems 15 – 17, expand the function as a Taylor series about the given value of *x*. Write enough terms to reveal clearly that you have seen the pattern.

15. about

16. about

17. about

18. Find the Maclaurin series for by equating derivatives. Compare the answer and the ease of finding the answer, with the series you obtain by substituting 3*x* for *x* in the original series.

19. Find the Maclaurin series for by equating derivatives. Compare the answer and the ease of finding the answer, with the series you obtain by substituting 1 + *x* for *x* in the Taylor Series for expanded about *x* = 1.

20. Estimate the using , fourth partial sum of the Taylor Series. How close is your answer to the exact answer? How does the error in the series value compare with the first term of the tail of the series, which is the first term left out in the partial sum.

21. Find the interval of values of *x* for which the fourth partial sum of the Taylor series for gives values that are within .0001 unit of .